

Integrating College-Preparatory Chemistry and Algebra II
Cary Kilner – University of New Hampshire - carypq@aol.com
ChemEd 2005 ; Vancouver, BC

Please study this list before the presentation. Choose those topics on which you wish to comment; that you are most interested in, most troubled by, or have worked with extensively in your classroom. After a brief introduction, I will display each topic in turn on the overhead followed by my diagnosis of the problem for your consideration. Then I will display how I have addressed, or suggest how it is possible to address, this problem.

I will set the egg timer for 5 minutes during which time you may discuss, criticize, argue, or propose alternatives. I will record this interchange and take notes, interjecting only when necessary for clarification. One of you will act as moderator to equitably select speakers for each topic. Please do not hog the floor when called upon; keep your discourse short so that others may respond to you. Select one or two topics you feel most strongly about or have the most experience with. There are 16 topic areas, so we will cover 8 topics during each session under this title. I welcome further interchange informally with you during the conference and via e-mail.

First we will consider common nomenclature that your math department may use, then examine a “chem-math” protocol with “rules,” principles and definitions for students to understand and follow, and look at my initial diagnostic quiz of math skills.

The 16 topics:

- 1) Rearranging Algebraic Expressions
- 2) Scientific Notation and Scaling
- 3) Metric vs. English measurement units
- 4) Unit Analysis (aka dimensional analysis/factor-label method)
- 5) Unitary-Rate Functions and Derived Units
- 6) Ratio/Proportional Reasoning
- 7) Correct rounding/Significant Figures/Precision of measurement
- 8) Percent, Percent-Composition, Percent Error (accuracy)
- 9) Scientific Graphing
- 10) Authentic Problem-Solving vs. Exercises
- 11) Mixture-Problems and Weighted-averages
- 12) More Scaling; Quadratic and Cubic Functions

- 13) Whole-number Logarithmic Relationships
- 14) Inverse-Square Relationships and Sign Conventions for Potential Energy
- 15) Rate-Problems
- 16) Complex Fractions

Course integration involves these considerations

- 1) Algebra II and Chemistry teachers use a different vocabulary when dealing with the same mathematical operations, sometimes confusing students across the two courses.
- 2) Algebra II and Chemistry teachers usually aren't aware of constructive correspondences in their syllabi, so they need to compare and talk!
- 3) Algebra II teachers may never have done engineering calculations, and have no idea how math is used in the physical sciences other than when they previously took these courses themselves. They treat mathematics as an abstract, proof-based subject, and measurements and applications may be secondary to them.
- 4) Conversely chemistry teachers may not be aware of the vast array of new methods of teaching mathematics which have arisen in the last two decades using technology. Although many teachers use the CBL probes for data collection and analysis, their methods are often very rudimentary compared to the sophisticated

work done in Algebra II with graphing calculators.

5) The NCTM has been out in front of the science disciplines in the standards movement, and science/chemistry teachers should read and study their publications.

Issues of Nomenclature

Use these phrases:

Don't use these:

System of equations

Two equations in two unknown, or simultaneous equations

Factor out units, or
Divide out units, or
Units that go to one.

Cancel units

List your inventory

Make a grocery list

Set up, distribute terms,
combine terms, and solve.

merely "solve"

Simplify

Rearrange

Direct and inverse

Direct and inverse

variation

proportion

Useful symbols to use:

Therefore

Exactly equal to

Approximately equal to

Defined as

Rounded to correct sig-figs

Proportional to

Chem-Math Protocol: Rules & Procedures

- 1) Your mechanics, not your answer, are most important to me. If your mechanics are correct, I know that you can obtain a correct result.
- 2) I must see your work clearly to appraise your mechanics.
- 3) The result from the calculator follows the equals sign, then an arrow tells me that you are rounding to correct significant figures with this final answer boxed in with units.
- 4) Use a dot or parentheses to show multiplication; an “x” denotes an unknown variable. In most cases you will already have a variable with a label, e.g., “T” for temperature, “P” for pressure, “V” for volume, “m” for mass, et.al.
- 5) Do not use slanted lines for divisors in your exercises. However they are okay for use in text

for ease in printing.

Three Chem-Math Rules:

- 1) Every number has a unit associated with it; it's a measurement of a physical quantity. The exception is a ratio, such as specific gravity, where the units will have factored out.
- 2) Every measurement must be correctly expressed. The decimal places convey information about the precision of the measuring instrument used.
- 3) Every calculation must be correctly rounded; your calculator will spew forth many extra digits. Follow the rules of significant figures.

Format for each slide:

Skill or topic area (see handout)

- A) Why needed or where it falls in the syllabus

- B) Pedagogical difficulty or particular problem it constitutes in the curriculum

C) Suggested handling or the way I have addressed these problems in my teaching

1) Rearranging Algebraic Expressions

A) This is an essential Algebra I skill.

B) Students can't solve for a divisor.
They can't divide by a fraction.
They forget their order of operations.
They forget the distribution law.
They forget the distributive properties.

- C) Write and administer a diagnostic quiz right away. Keep track of scores and compare with end-of-year grades, especially if you have leveled courses.

Do lots of drill and remediation.

Practice rearranging chemistry math-formulas before students encounter them, using the symbols they will be using. Point out that symbols for various quantities may be different in various texts and with various teachers.

Derive cross-multiplication with your students for their future use, demonstrating the correct algebra used.

Give examples to the math department.

2) Scientific Notation & Scaling I

- A) This is an essential tool of measurement.

Scaling is a *Big-Idea* found in the NSES.

- B) Students don't understand "orders of magnitude," therefore they don't understand scientific notation applications,

such as the concept of exponential scaling, such as in correctly drawing a Geological/Biological Time-Line. Students are unsure of their Laws of Exponents, or cannot transfer them from abstract math to science applications. Students do not understand standard-form, and insist upon writing out the calculator display, using “E-notation.”

- C) Teach your students standard-form and do not allow them to write E-notation. Tell them “when you put a number into standard-form-if you make the number smaller-use a larger power of ten,” and vice-versa, then review negative exponents. Review the Law of Exponents. Give lots of drill and practice. Communicate with your kindred spirit in the math department; give her some examples of computations using, for example, N_A and h (Planck’s constant). Show the *Powers of Ten* video in both classes.

3) Metric vs. English measurement units

- A) The use of the SI system, and a feel for Metric measure, is a rite-of-passage into all the sciences.
- B) Our students have no intuitive feel for Metric measure because they have grown up using English units.
- C) Don't enforce "Metric-only" rules; allow students to compare equivalent units. Have them do introductory measurement activities of weight (force), length, temperature, and volume (even though these should have been done in earlier science courses).
Have students develop rule-of-thumb approximations for common conversion factors, and have them compile (or give them) a table of conversion factors they will use in their upcoming chemical calculations. See your kindred spirit in the math department and exchange activities and exercises.
See the 9th grade Physical Science and Algebra I teachers and urge them to incorporate extensive measurement concepts and activities into their curricula.

4) Unit-Analysis

A) UA is a useful algorithm for doing scientific and engineering work.

It is vital for understanding and using derived units.

It directs the computational process.

Using multistep UA minimizes the accumulation of rounding errors.

B) Many students learn UA as a strictly mechanical process without understanding the mathematics of why it works.

Other students have continual difficulty with UA throughout the year and never use it correctly.

Students use slanted lines, which gradually degenerate into vertical lines so they lose track of the denominators, thereby missing a central purpose of the process.

Some assert unequivocally that they “can’t do math,” and sabotage any attempts to show them how to perform chemistry computations correctly.

C) Introduce UA early on and practice daily.

Use nonsense variables as well as authentic exercises.

Always require units with measurements.
Ensure your math department is on board
and using UA correctly.

5) Unitary-rate Functions and Derived Units

A) These define intensive properties. They also show the source of the variable.

B) Students don't know what "per" means.
Students use slanted lines that degenerate into vertical lines.
Students can't divide by a fraction correctly, which persists into kinematics as in deriving acceleration.

C) Provide a list of example linear unitary-rate functions from physics and chemistry.
Have students select a function that demonstrates direct variation.

Ask them to select and rearrange a function to demonstrate inverse variation.

Give them some data and have them graph these two examples. Show them that the slope becomes the unitary-rate constant being defined or calculated.

Have them do a cumulative data collection

activity with a substance where they add increments of mass and measure the accumulating volume, to determine the density. Graph it.

Discuss unitary-rates with the math department.

6) Ratio/Proportional Reasoning

A) This skill represents formal reasoning in Piagetian terms.

It is the basis for all of quantitative chemistry.

B) Students don't understand "lowest whole-number ratio."

Students can't distinguish between direct and inverse variation (see previous slide).

Students can't write a mathematical function that represents a physical reality.

C) Reasoning with proportionality is a different skill from calculating with UA.

Use simple systems to teach proportionality on a gradient: first density, then percent-composition, then perhaps concentration, then the three gas-laws.

Give lots of examples to work within and without chemistry.

(Let students use ratio/proportion for solving stoichiometry problems only if they can use UA first.)

Find out how the math department addresses proportionality. Check out the NCTM standards for its various locations in the K-12 math continuum.

- 7) Correct rounding; Significant Figures; Precision of measurement
- A) A vital concept and *Big-Idea* in the NSES
 - B) Students mindlessly report all digits from their calculators. They invariably drop zeros after a decimal. They think every measurement and calculation is exact. They confuse the everyday use of “precision” with its technical use; the same goes for “accuracy.”
 - C) Many students will not understand the use of sig-figs at first; they need repeated exposure. *Times through the material equals certainty*, means give them lots of practice. *Duplication precedes understanding*, means that if they don't understand the rules listed in their textbook, have them refer to the

rules whenever they compute and it will come. Teach the rules for expression in one sitting, and the rules for rounding in another. Tell them to “round to the least number of digits” used in a problem and it will work most of the time. Give them a worksheet with a handful of ambiguous examples and refer to it often.

Design activities to practice measurements and calculations of derived quantities.

Get the Math Department on board by doing lots of measuring activities!

8) Percent, Percent-Composition, Percent-Error

A) Important for overall numeracy

B) Students don't understand what “percent” means.

They don't know what a “decimal-fraction” is, therefore they don't understand a mass-fraction or a volume-fraction.

They don't understand that “percent-composition” means the percent of each constituent or element in a mixture or a compound.

They use a calculator to move the decimal.

They can't get the whole when given a part and a percent.

- C) Teach that “percent” means parts-per-hundred. Teach that “%” is a unit. It means pph. Have them use scientific notation in this manner: $x\% = \text{decimal-fraction} \cdot 10^2\%$
Then define ppm, ppb, & ppt similarly, using scientific notation.

Teach percent-error as accuracy, and as a comparison. Use absolute value to show it as the deviation from an expected experimental result.

Talk to the Math folks about how they use percent. See the NCTM standards regarding the teaching of statistics.

9) Scientific Graphing

- A) This is a cumulative skill, very cross disciplinary, and an essential aspect of numeracy.

- B) Students draw on autopilot from their math classes. They like to do this.

They don't label their axes.

They don't title their graphs.

They use pen and make a mess.

They don't understand that the slope constitutes a unitary-rate function.

They can't identify the dependent and the independent variable (although they can robotically define them).

They can't scale their axes effectively or correctly.

They don't know what residuals are when their TI-83+ computes the curve-of-best-fit.

C) Practice with lots of real examples.

Give them a strict list of graphing rules.

Get with the Math teacher and give her some physical science examples to use.

Design joint exercises and activities to perform and to graph in both classes.

Find out how the Math Department uses the TI-83+ in their instruction.

10) "Authentic problem-solving"

A) This is the critical-thinking skill sought by all science educators (formal reasoning).

"A 'problem' is what you do when you don't know what to do." (G.Bodner)

B) Students can master algorithms, but then are unable to apply them in novel settings.

They cannot see the stages in a multi-step word-problem and bog down easily.

They can't filter out the "noise" in word problems.

They don't understand "chunking" (information-sequencing).

They don't understand "guestimation," order-of-magnitude approximations of answers (aka "back-of-the-envelope calculations" by engineers).

They get frustrated easily and are unwilling to start over and try other stuff out.

They confuse units unless you define them clearly, e.g. Q vs. C_p vs. q vs. ΔH ; K_{eq} vs. the rate-law k vs. the Beer's Law k .

- C) Design a gradient of real-life transfer problems that unfolds throughout the year so students can gradually develop confidence, facility and expertise. An example could be to obtain a desired dilution of a concentrated acid, given percent acid and density of concentrate. Have a colloquium with the Math Department on problem solving versus exercises.

11) Mixture-Problems

- A) This constitutes a large segment of transfer

problems, and relies on proportional reasoning. Examples are:

The price of trail-mix (gorp),

The composition of a 2-component alloy,

Isotopic composition and atomic weight,

Dilution problems,

Calorimetry problems,

Marathon stoichiometry problems.

B) Students don't see the proportional relationships that are occurring.

They don't understand the concept of weighted-averages.

They don't see how they can use their matrix function to solve these.

They mess up the algebra and give up.

They accept negative (nonsense) values.

C) Develop this skill gradually throughout the year to demonstrate its universal application.

Teach the four ways to consider the mathematics of mixture-problems:

numerical, graphical, algebraic, and verbal.

Discuss these with the Math Department.

12) Scaling II; quadratic and cubic functions

- A) Here is another bridge between Chemistry and Physics, and Algebra II. It constitutes an important connection across disciplines.
- B) Students don't understand the "pizza-problem," i.e., they don't understand exponential growth.
They see too few examples and too sporadically, since these are not grouped together in the syllabus.
- C) Develop a unit on this concept.
Do lots of examples: the satellite dish/size of telescope problem, heating a large vs. a small building and, from Biology, the area and volume of a cell vs. its radius (i.e. the rate of diffusion of nutrients and waste products across the cell membrane vs. the total mass of nutrients needed for the cell).
Calculate the force crushing the can, and holding the Magneburg Spheres together.
Get with your Math person and share ideas.

13) Logarithmic Relationships

- A) The magnitude of stars
The Richter scale
The decibel scale
The pH scale

- B) Students don't transfer understanding from their Algebra II course to these relationships. They perform their calculations mechanically without understanding how the underlying mathematics relates to what they have studied in Algebra II.

- C) Reteach or review up front as necessary.
Show the *Powers of Ten* video again.
Make sure students are totally comfortable with using whole-number logs before moving on to decimal logs.
Review the Law of Exponents.
Coordinate with your Math teacher.

14) Inverse-Square Relationships, and Sign Conventions for Potential Energy

- A) Newton's Law of Universal Gravitation, "candle problems," Coulomb's Law, and Graham's Law.
- B) While students may be facile with this function as a math abstraction in Algebra II, they do not transfer it conceptually to these applications.

In U-Grav, E_p goes from 0 to a positive number at infinity, whereas in atomic ionization energies, E_p goes from a minimum negative number to 0 at infinity.

- C) Clarify these conventions for your students; no one else will compare and contrast these two applications unless you do. Design a pictorial worksheet to help them. Have students graph the functions, and in the correct quadrant.

Review the physics: derive work from

$F = ma$ and show where the units and sign conventions come from.

Have your students do the classic Graham's Law demonstration for themselves, derive the law, then compare data.

Write practice problems for them to work.

Share your ideas with the Algebra II teacher.

15) Rate-Problems

A) These are exponential decay functions: radio-dating/half-life problems, Newton's Law of Cooling, Beer's Law, rinse problems, and first-order kinetics. They require more than a superficial understanding of the mathematics, even when using an algorithm.

B) Students are not asked to derive these laws using experimental observations (or tabulated data) and their learned abstract mathematics.

Students don't recognize an asymptotic relationship; they want the curve to hit the axis somewhere.

C) This is a truly rich place to integrate math and science.

Do a cross-section of problems from the above list so that students understand the mathematics before looking at the chemistry.

Use small group work and have students derive the necessary formulae to use.

Have an extended discussion with your kindred spirit in the math department.

16) Complex Fractions

A) These are found in the Rydberg equation, the Clausius-Clapeyron equation, in calculating the reduced mass of a system, and in finding resistance and capacitance in series and parallel circuits.

B) Although students learned complex fractions in Algebra I, they have not needed the skill up to this point so there is a lot of time intervening.

They will need this skill shortly in physics and electricity.

- C) Find out what examples your Algebra I teachers use and discuss how they teach complex fractions.
Obtain a worksheet from them, with only the math exercises to practice, before going on to the chemistry (or physics).
Share your applications with them so they can use them when teaching complex fractions themselves.